

Dark matter on the lattice

Randy Lewis

Department of Physics and Astronomy, York University, Toronto, Ontario, Canada M3J 1P3

Abstract. Several collaborations have recently performed lattice calculations aimed specifically at dark matter, including work with SU(2), SU(3), SU(4) and SO(4) gauge theories to represent the dark sector. Highlights of these studies are presented here, after a reminder of how lattice calculations in QCD itself are helping with the hunt for dark matter.

Keywords: lattice gauge theory, dark matter, lattice quantum chromodynamics

PACS: 11.15.Ha, 95.35.+d, 12.38.Gc

During the past 3 years, there have been exploratory lattice studies of dark matter with an SU(2) gauge theory [1, 2, 3, 4, 5], an SU(3) gauge theory [6], an SU(4) gauge theory [7] and an SO(4) gauge theory [8]. Before presenting these, let's recall how lattice QCD has determined a matrix element that is valuable to a broad range of dark matter models.

LATTICE QCD STUDIES FOR DARK MATTER

For a valuable review of this topic, including many details and references to the extensive original literature, see Ref. [9] by Junnarkar and Walker-Loud. The discussion presented below is merely an appetizer.

If dark matter is a weakly-interacting massive particle (WIMP), then experimental observation requires knowledge of how the WIMP interacts with a detector. Therefore the basic WIMP-nucleon interaction is of fundamental interest. Although the microscopic theory of WIMPs is unknown, the low-energy limit of a generic spin-independent interaction is scalar and that scalar could couple to any flavor of quark in the nucleon (not only valence flavors). Up/down quark contributions can be extracted from pion-nucleon scattering experiments, charm/bottom/top quark contributions are amenable to perturbation theory, and the strange quark contribution has been obtained from lattice QCD calculations.

There are no valence strange quarks in a nucleon and it is notoriously difficult to calculate with lattice QCD in a situation like this one, where the scalar probe is coupling to a virtual quark rather than a valence quark. Nevertheless, many collaborations have accepted the challenge [10]. The matrix element studied in lattice QCD is

$$f_s = \frac{\langle N | m_s \bar{s}s | N \rangle}{m_N} = \frac{m_s}{m_N} \frac{\partial m_N}{\partial m_s} \quad (1)$$

where the first equality defines the dimensionless ratio f_s and the second equality is the Feynman-Hellman theorem.

Click the box to see Fig. 8 from Ref. [9], showing 17 determinations of f_s from the lattice QCD community.

http://inspirehep.net/record/1209574/files/figures_fs_compare.png

The lattice QCD implementations differ from one another in several ways, and this provides confidence in understanding systematic effects. Some calculations use the Feynman-Hellman theorem while others calculate the matrix element directly. Different lattice spacings, lattice volumes and lattice actions are used. Different methods of reaching the physical up/down masses are employed. The color coding in the figure reflects the confidence that Junnarkar and Walker-Loud recommend for individual determinations of f_s but, regardless of these colors, the lattice results display a clear indication that f_s is significantly smaller than 0.1. The authors of Ref. [9] arrive at a lattice average of

$$f_s = 0.043 \pm 0.011 \quad (2)$$

which is valuable input for phenomenological studies of dark matter.

PUTTING DARK MATTER DIRECTLY ON THE LATTICE

Dark matter is physics beyond the standard model. If that physics comes with a new strong interaction, then lattice field theory could be a valuable calculational tool. Each section below discusses lattice studies performed for a particular candidate gauge theory in the dark sector. The discussion here will be very brief, and readers should consult Refs. [1, 2, 3, 4, 5, 6, 7, 8] for important details and for references to the large body of phenomenological literature on which many of the underlying ideas are based.

SU(2) gauge theory

As a smaller cousin of QCD, SU(2) is a natural starting point for lattice explorations of possible gauge theories in the dark sector. A convenient matter content is provided by having just 2 Dirac fermions, both in the fundamental representation. As verified by the explicit lattice calculations in Ref. [1], this theory has a global SU(4) symmetry that is dynamically broken to Sp(4), and the corresponding 5 Goldstone bosons are observed in the spectrum. They couple to the following operators corresponding to 3 dark pions, 1 dark nucleon and 1 dark anti-nucleon, which are all spinless in SU(2):

$$\Pi^+ \Leftrightarrow \bar{U} \gamma_5 D, \quad (3)$$

$$\Pi^- \Leftrightarrow \bar{D} \gamma_5 U, \quad (4)$$

$$\Pi^0 \Leftrightarrow (\bar{U} \gamma_5 U - \bar{D} \gamma_5 D) / \sqrt{2}, \quad (5)$$

$$\Pi_{UD} \Leftrightarrow U^T (-i\sigma^2 C) \gamma_5 D, \quad (6)$$

$$\Pi_{\bar{U}\bar{D}} \Leftrightarrow \bar{U} (-i\sigma^2 C) \gamma_5 \bar{D}^T, \quad (7)$$

where C is the charge conjugation matrix and the Pauli structure $(-i\sigma^2)$ acts on color indices. If the U and D fermions have equal non-zero masses, then all 5 (now pseudo) Goldstone bosons acquire a common mass.

Click the box to see Fig. 4 from Ref. [1], showing the Goldstone boson mass squared versus fermion mass.

<http://inspirehep.net/record/927706/files/GBmass.png>

The left panel of the figure confirms the expected $m_\Pi^2 \propto m_q$ behavior for Goldstone bosons. The right panel confirms the expected deviations due to finite volume and due to large m_q .

Where would these 5 Goldstone bosons appear in nature? Ref. [1] mentions the option that the 3 dark pions could be eaten by the W^\pm and Z bosons while the dark nucleon is a dark matter candidate and the dark anti-nucleon has been annihilated away through baryon asymmetry (like the QCD anti-nucleon). This option would require the 125 GeV boson to emerge from the SU(2) theory as a 0^{++} hadron, and corresponds to technicolor-style electroweak couplings.

Ref. [2] takes a more general view, allowing for an arbitrary rotation angle θ somewhere between the technicolor limit ($\theta = \pi/2$) and a composite Higgs limit ($\theta = 0$). In the latter limit, the dark nucleon/anti-nucleon degrees of freedom realign into a composite Higgs and a dark matter candidate.

Beyond the pseudo-Goldstone bosons, this SU(2) theory has vector and axial-vector mesons awaiting experimental discovery. Ref. [2] computes the vector mass to be

$$m_\rho = \frac{2.5 \pm 0.5 \text{ TeV}}{\sin \theta} \quad (8)$$

based on data in the following figure:

Click the box to see Fig. 5 from Ref. [2], showing the vector and axial-vector meson masses versus fermion mass.

http://inspirehep.net/record/1289883/files/spectrum_2.png

Ref. [3] calculates the cross section for scattering the dark matter candidate from a proton. Contributions from photon exchange and from composite Higgs exchange are both included but, as we'll see below, the photon dominates when approaching current experimental bounds.

A direct lattice calculation of the photon coupling to each of the 5 Goldstone bosons is given by

$$C_{UD}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = T^U - T^D, \quad (9)$$

$$C_{\overline{UD}}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = -T^U + T^D, \quad (10)$$

$$C_{\Pi^+}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = T^U + T^D, \quad (11)$$

$$C_{\Pi^-}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = -T^U - T^D, \quad (12)$$

$$C_{\Pi^0}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = 0, \quad (13)$$

where

$$T^X = \sum_{\vec{x}_i, \vec{x}, \vec{x}_f} e^{-i(\vec{x}_f - \vec{x}) \cdot \vec{p}_f} e^{-i(\vec{x} - \vec{x}_i) \cdot \vec{p}_i} \langle 0 | \Pi(t_f, \vec{x}_f) V_\mu^X(t, \vec{x}) \Pi^\dagger(t_i, \vec{x}_i) | 0 \rangle. \quad (14)$$

Eq. (9) vanishes if $m_U = m_D$, so the dark matter candidate has no electromagnetic form factor at all in that limit. Unfortunately, a lattice calculation with $m_U \neq m_D$ is very costly due to the presence of disconnected quark loop effects. Ref. [3] avoids this dilemma by using lattice simulations at $m_U = m_D$ together with vector meson dominance. A priori it is not known whether vector meson dominance is reliable in an SU(2) gauge theory, so the lattice calculations in Ref. [3] were used to verify its applicability.

Click the box to see Fig. 9 from Ref. [3], showing the applicability of vector meson dominance for these lattice data.

<http://inspirehep.net/record/1249877/files/ff3.png>

The photon couples to the dark matter candidate through the charge radius, which in this case is

$$\mathcal{L}_B = ie \frac{d_B}{m_\rho^2} \phi^* \overleftrightarrow{\partial}_\mu \phi \partial_\nu F^{\mu\nu}, \quad \text{where } d_B = \frac{m_{\rho U} - m_{\rho D}}{m_\rho}. \quad (15)$$

Notice that the charge radius vanishes when $m_U = m_D$, as expected. The cross section obtained from photon exchange and composite Higgs exchange with $|d_B| = 1$ and $|d_B| = 0.1$ (dot-dashed in the figures) is essentially proportional to $|d_B|^2$, indicating that composite Higgs exchange is negligible. The cross section can be large enough to be constrained by upcoming experimental searches.

Click the boxes to see Fig. 10 from Ref. [3], showing the dark matter-proton cross section and experimental bounds.

http://inspirehep.net/record/1249877/files/pNGB_DM2.png

http://inspirehep.net/record/1249877/files/pNGB_DM1.png

Ref. [5] calculates the scalar coupling to some of the dark hadrons in this SU(2) theory. It is the analogue of Eq. (1) but now in the dark sector, i.e.

$$f_q^{(H)} = \frac{\langle H | m_q \bar{q} q | H \rangle}{m_H} = \frac{m_q}{m_H} \frac{\partial m_H}{\partial m_q} \quad (16)$$

where H denotes a dark hadron of interest. Several calculations are done for a modest range of valence fermion masses surrounding the sea quark mass, and the Feynman-Hellman theorem is used to arrive at precise numerical results. The authors of Ref. [5] expect $O(30\%)$ systematic errors due to partial quenching.

Click the box to see Fig. 26 from Ref. [5], showing the determination of f_{u+d}^H for 2 hadron multiplets.

http://inspirehep.net/record/1300660/files/figures_fHPlot.png

Refs. [4, 5] point out that, like QCD, the SU(2) theory could also have a nuclear physics spectrum with several nuclei contributing to dark matter. Lattice calculations are used to test several options, learning whether they form nuclear bound states or are merely scattering states. The basic idea is to study the dependence of energy on lattice volume L^3 ,

$$\Delta E(L) \propto \frac{1}{L^3} + \dots \quad \Rightarrow \quad \text{scattering states} \quad (17)$$

$$\Delta E(L) = -\frac{\gamma^2}{2\mu} \left(1 + \frac{12\hat{C}}{\gamma L} e^{-\gamma L} \right) \quad \Rightarrow \quad \text{bound states} \quad (18)$$

but please consult Ref. [5] directly for a thorough discussion. Detmold, McCullough and Pochinsky find evidence for bound states with $J^P = 1^+$ composed of $N\Delta$ and $NN\Delta$ and perhaps also $NNN\Delta$. Recall that all hadrons are bosons in a 2-color theory; N and Δ denote spin 0 and spin 1 states respectively. This study chooses fermion masses that lead to $m_\rho/2 < m_\pi < m_\rho$ in the dark sector rather than working near the chiral limit. As in Refs. [1, 2, 3], the overall scale is set by $f_\pi = 246$ GeV.

Click the boxes to see Fig. 23 from Ref. [5], showing the determination of γ for $N\Delta$, $NN\Delta$ and $NNN\Delta$.

http://inspirehep.net/record/1300660/files/figures_TeV2DgammaFit-1NDelta.png

http://inspirehep.net/record/1300660/files/figures_TeV2DgammaFit-2NDelta.png

http://inspirehep.net/record/1300660/files/figures_TeV2DgammaFit-3NDelta.png

SU(3) gauge theory

The lattice community has a lot of experience with SU(3) gauge theory for QCD. Ref. [6] considers the possibility of a new SU(3) gauge theory in the dark sector having either 2 or 6 flavors of Dirac fermions in the fundamental representation. All dark fermions are taken to be singlets under the standard model weak interaction and to come in pairs with electric charges $Q_U = +2/3$ and $Q_D = -1/3$. In contrast to the SU(2) case discussed above, this new SU(3) sector is not given any role in electroweak symmetry breaking. The $N_f^2 - 1$ Goldstone bosons are assumed unstable, leaving the lightest baryon (i.e. the dark neutron) as the dark matter candidate.

A lattice calculation of the electromagnetic form factor is used to obtain the event rate that would be observed in the XENON100 experiment, as a function of the dark matter particle's mass. Disconnected fermion loops, presumably a small contribution, are omitted because of their excessive expense. The authors of Ref. [6] observe that the mean square charge radius, $\langle r_E^2 \rangle$, is an order of magnitude larger than its QCD counterpart due to their use of a quite large mass for the dark fermion.

Click the box to see Fig. 6 from Ref. [6], comparing the XENON100 bound to a specific SU(3) dark matter model.

http://inspirehep.net/record/1209894/files/rate_XE100_6p6-43p3_12075988_full_rEonly.png

In the figure, solid curves are the computed event rate, while dashed curves show the contribution from just the charge radius term. Notice that the cross section has only a tiny sensitivity to whether $N_f = 2$ or 6. To respect the bound from XENON100, the dark matter particle must have a mass of more than 10 TeV.

SU(4) gauge theory

Ref. [7] reports on a quenched exploration of an SU(4) model for dark matter. Like the SU(2) case, all hadrons are bosons. Unlike the SU(2) case, all hadrons do not have just 2 valence fermions; mesons still have 2 but baryons now have 4.

A first step is to compare the mass spectrum of SU(4) [with solid error bars] to the more familiar SU(3) [with dashed errors bars]. Two options for the fermion mass are displayed, and the horizontal axis shows that the mass ratio of pseudoscalar to vector mesons is in the 70%-80% range. The authors note that the pseudoscalar mass must be larger than 100 GeV to satisfy LEP bounds.

The lattice results show that meson masses are largely independent of whether $N_c = 3$ or 4, as expected. The lattice results also show that baryon masses [$J = 0, 1, 2$ for SU(4) and $J = 1/2, 3/2$ for SU(3)] are roughly proportional to N_c , as expected.

Click the box to see Fig. 3 from Ref. [7], comparing the hadron masses in SU(3) and SU(4) gauge theories.

http://inspirehep.net/record/1282609/files/3cv4c_no_rotor.png

Ref. [7] calculates the dark matter-nucleon scattering cross section as a function of the coupling of the dark baryon to the Higgs, which is named α ,

$$\alpha = \frac{v}{m_f} \left. \frac{\partial m_f(h)}{\partial h} \right|_{h=v}, \quad \text{where } v = 246 \text{ GeV}. \quad (19)$$

The following figure shows that, for one particular value of m_{PS}/m_V , the coupling α must be quite small to be compatible with the LUX experimental bound. LEP constraints require $m_B > 300$ GeV. Effects of lattice spacing, lattice volume, and a broader range of m_{PS}/m_V were also studied; please consult Ref. [7] for more information.

Click the box to see Fig. 5(b) from Ref. [7], showing a dark matter-nucleon cross section in an SU(4) gauge theory.

<http://inspirehep.net/record/1282609/files/LUX-exclusion-k015625-label.png>

SO(4) gauge theory

The first lattice study applying a non-SU(N) theory to dark matter is Ref. [8], authored by Hietanen, Pica, Sannino and S ndergaard. They use 2 (Wilson) Dirac fermions in the vector representation and begin by exploring the phase diagram through varying the 2 parameters: bare inverse gauge coupling β and bare fermion mass m_0 . A physical region is found where β and m_0 are both sufficiently large.

Click the box to see Fig. 2 from Ref. [8], showing the phases of a lattice SO(4) gauge theory.

<http://inspirehep.net/record/1203463/files/LatticePhases.png>

The Polyakov loop is a gauge-invariant path in one spatial direction, closing back on itself due to periodic boundary conditions. Since the lattice action treats every spatial direction equally, Polyakov loops are expected to be statistically equivalent in each spatial direction. Ref. [8] observed an interesting counterexample in small lattice volumes, interpreted as the emergence of two distinct phases during the simulation. This phenomenon was not observed for larger volumes.

Click the box to see Fig. 6 from Ref. [8], showing spatial Polyakov loops for a $12^3 \times 64$ lattice.

http://inspirehep.net/record/1203463/files/L12T64beta7m03_pit.png

This theory has a global $SU(4)$ symmetry that breaks to a global $SO(4)$, producing 9 Goldstone bosons. The Goldstone boson having isospin zero is the dark matter candidate. The pseudoscalar, vector and axial-vector meson masses have been calculated as a function of quark mass, and the Goldstone nature of the pseudoscalars has been observed.

Click the box to see Fig. 8 from Ref. [8], showing meson masses in an $SO(4)$ gauge theory.

<http://inspirehep.net/record/1203463/files/L24T64b7mesons.png>

Click the box to see Fig. 9 from Ref. [8], showing the vector to pseudoscalar mass ratio for an $SO(4)$ gauge theory.

http://inspirehep.net/record/1203463/files/L24T64b7PS_V_ratio.png

SUMMARY

Studying dark matter models with lattice field theory is a recent effort comprising Refs. [1, 2, 3, 4, 5, 6, 7, 8] and there is a lot more that can be explored. The most-studied model so far is $SU(2)$ gauge theory with 2 Dirac fermions in the fundamental representation [1, 2, 3, 4, 5] that connects dark matter to electroweak symmetry breaking.

The lattice QCD determination of $\langle N | m_s \bar{s} s | N \rangle$ is mature [9, 10], with independent studies by many collaborations and detailed accounting of systematic uncertainties.

ACKNOWLEDGMENTS

Thanks to the organizers of *XIth Quark Confinement and the Hadron Spectrum* for the opportunity to participate. Thanks to Ari Hietanen, Claudio Pica and Francesco Sannino for valuable collaboration into dark matter on the lattice. This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

REFERENCES

1. R. Lewis, C. Pica and F. Sannino, Phys. Rev. D **85**, 014504 (2012) [arXiv:1109.3513].
2. A. Hietanen, R. Lewis, C. Pica and F. Sannino, JHEP **1407**, 116 (2014) [arXiv:1404.2794].
3. A. Hietanen, R. Lewis, C. Pica and F. Sannino, arXiv:1308.4130.
4. W. Detmold, M. McCullough and A. Pochinsky, arXiv:1406.2276.
5. W. Detmold, M. McCullough and A. Pochinsky, arXiv:1406.4116.
6. T. Appelquist *et al.* [Lattice Strong Dynamics (LSD) Collaboration], Phys. Rev. D **88**, 014502 (2013) [arXiv:1301.1693].
7. T. Appelquist *et al.* [Lattice Strong Dynamics (LSD) Collaboration], Phys. Rev. D **89**, 094508 (2014) [arXiv:1402.6656].
8. A. Hietanen, C. Pica, F. Sannino and U. I. Søndergaard, Phys. Rev. D **87**, 034508 (2013) [arXiv:1211.5021].
9. P. Junnarkar and A. Walker-Loud, Phys. Rev. D **87**, 114510 (2013) [arXiv:1301.1114].
10. R. D. Young and A. W. Thomas, Phys. Rev. D **81**, 014503 (2010) [arXiv:0901.3310];
D. Toussaint *et al.* [MILC Collaboration], Phys. Rev. Lett. **103**, 122002 (2009) [arXiv:0905.2432];
K. Takeda *et al.* [JLQCD Collaboration], Phys. Rev. D **83**, 114506 (2011) [arXiv:1011.1964];
S. Dürr *et al.*, Phys. Rev. D **85**, 014509 (2012) [arXiv:1109.4265];
R. Horsley *et al.* [QCDSF-UKQCD Collaboration], Phys. Rev. D **85**, 034506 (2012) [arXiv:1110.4971];
G. S. Bali *et al.* [QCDSF Collaboration], Phys. Rev. D **85**, 054502 (2012) [arXiv:1111.1600];
S. Dinter *et al.* [ETM Collaboration], JHEP **1208**, 037 (2012) [arXiv:1202.1480];
A. Semke and M. F. M. Lutz, Phys. Lett. B **717**, 242 (2012) [arXiv:1202.3556];
M. Gong *et al.* [xQCD Collaboration], PoS LATTICE **2011**, 156 (2011) [arXiv:1204.0685];
W. Freeman *et al.* [MILC Collaboration], Phys. Rev. D **88**, 054503 (2013) [arXiv:1204.3866];
P. E. Shanahan, A. W. Thomas and R. D. Young, Phys. Rev. D **87**, 074503 (2013) [arXiv:1205.5365];
H. Ohki *et al.* [JLQCD Collaboration], Phys. Rev. D **87**, 034509 (2013) [arXiv:1208.4185];
X.-L. Ren, L. S. Geng, J. Martin Camalich, J. Meng and H. Toki, JHEP **1212**, 073 (2012) [arXiv:1209.3641];
M. Engelhardt, Phys. Rev. D **86**, 114510 (2012) [arXiv:1210.0025];
C. Jung [RBC and UKQCD Collaborations], PoS LATTICE **2012**, 164 (2012) [arXiv:1301.5397].